Math IV

[BSMA2002] (semester) / [BTMA2001] (tetramester)

Teaching notes addressed to the teacher

Tools

To ensure that you get the most out of your educational experience in this certificate mode, we recommend you review these tutorials.

Notes for the teacher corresponding to the explanation of Topic 1

Explain the properties of inequalities and demonstrate their application in the solution of inequalities.

Describe set notation, interval notation and the graphical representation of inequalities; make sure that the learner is able to identify and use the symbology of each of these notations.

Use an everyday life situation, preferably an example that is familiar to the learner to exemplify a relation between two variables and from that example guide the learner to identify the independent variable and the dependent variable, based on this you can point out the domain and range of the relation.

Point out the difference between a relation and a function and exemplify the different ways in which they can be represented: Venn diagram, set of ordered pairs and by means of a graph. It is important that you explain the various ways to differentiate in each of these formats if it is a relation or a function.

Explain the difference between the codomain and the range or image of a function. It is recommended that you define the meaning of a correspondence rule through an example and show the classification of the correspondence rules.

Notes for the teacher corresponding to Exercise 1

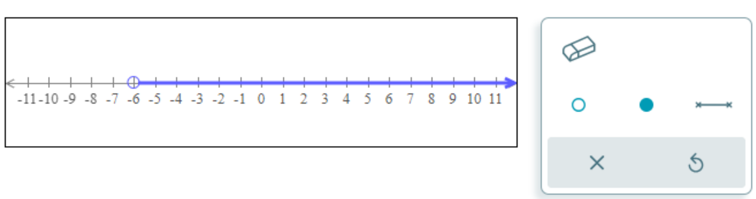
Before the explanation it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Graphing a linear inequality on the number line

Graph the inequality below on the number line.

$a > -6$



Make sure to explain the symbology of set notation and graphic notation.

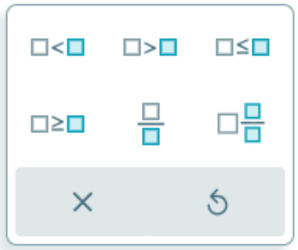
Solving a two-step linear inequality: Problem type 2

Solve the inequality for x.

$$-31 < 2x - 19$$

Simplify your answer as much as possible.

$-6 < x$

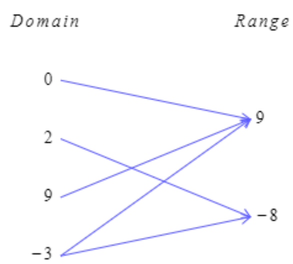


In this exercise the learner will apply the properties of inequalities, point out that he must simplify his answer.

Identifying functions from relations

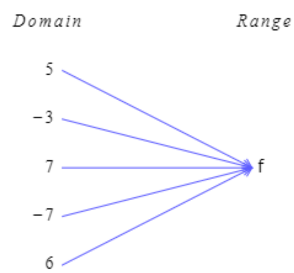
For each relation, decide whether or not it is a function.

Relation 1



- Function
- Not a Function

Relation 2



- Function
- Not a Function

Relation 3

{(9, g), (-6, g), (-2, g), (-5, d)}

- Function
- Not a Function

Relation 4

{(9, -8), (-8, -8), (9, 9), (-8, 9)}

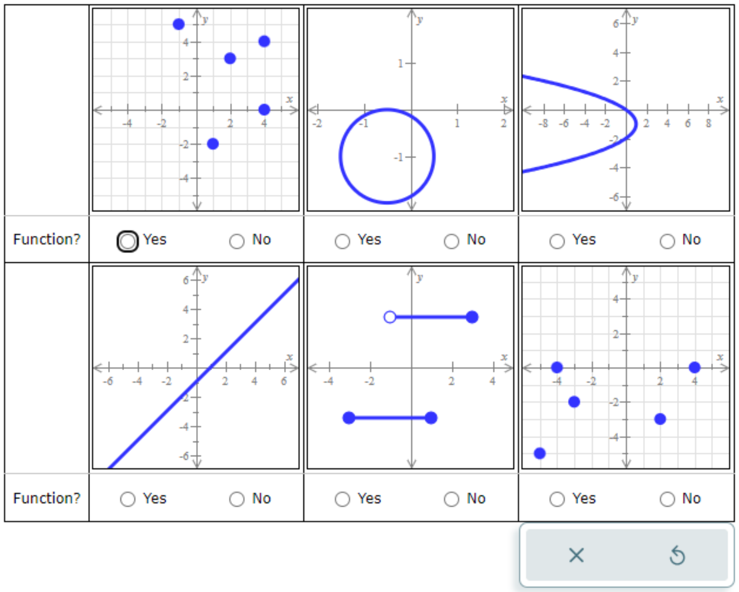
- Function
- Not a Function

Learner will identify whether the relation is a function or not.

Universidad Tecmilenio

Vertical line test

For each graph below, state whether it represents a function.



Learner will apply the vertical line rule to check whether the example is a function or not.

Table for a linear function

The function g is defined by the following rule.

$$g(x) = 4x + 3$$

Complete the function table.

x	$g(x)$
-3	
0	
1	
2	
5	

Invite the learner to identify which is the dependent variable and which is the independent variable.

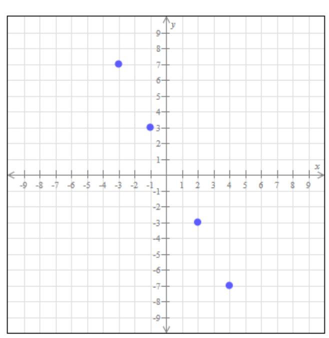
Graphing an integer function and finding its range for a given domain

The function h is defined as follows for the domain given.

$$h(x) = 1 - 2x, \quad \text{domain } = \{-3, -1, 2, 4\}$$

Write the range of h using set notation. Then graph h.

range = $\{-7, -3, 3, 7\}$



Domain and range of a linear function that models a real-world situation

A construction crew is lengthening a road. Let L be the total length of the road (in kilometers). Let D be the number of days the crew has worked. Suppose that $L = 4D + 300$ gives L as a function of D. The crew can work for at most 70 days.

Identify the correct description of the values in both the domain and range of the function. Then, for each, choose the most appropriate set of values.

	Description of Values	Set of Values
Domain	○ 1,length of the road (in kilometers) ◉ 2,number of days the crew has worked	the set of all real numbers from 0 to 70
Range	◉ 1,length of the road (in kilometers) ○ 2,number of days the crew has worked	the set of all real numbers from 300 to 580

Guide the learner to determine which variable depends on the other.

Notes for the teacher corresponding to the explanation of Topic 2

Return to the theme of the straight line and direct the learner to recognize when a graph is increasing, decreasing or constant.

Show a graph that presents the three types of behaviors and demonstrate to the learner how the graph is defined in one way or another for a given interval, indicate the behavior of the variables *x* and *y* in each case.

Explain to the learner that some functions are called special because of the properties that characterize them and present examples of each.

Analyze with the learner the transformations that the graph of a function can undergo through translation, reflection and rotation. Make sure the learner algebraically identifies these transformations in the function.

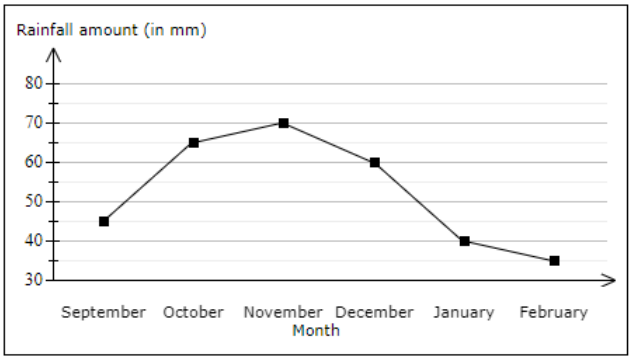
Notes for the teacher corresponding to Exercise 2

Before the explanation it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Interpreting a line graph

The graph below shows the amounts of rainfall for six months.

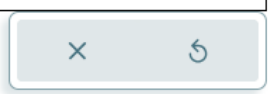


(a) What was the least amount of rainfall in a month?

☐ mm

(b) When did the greatest increase in rainfall occur?

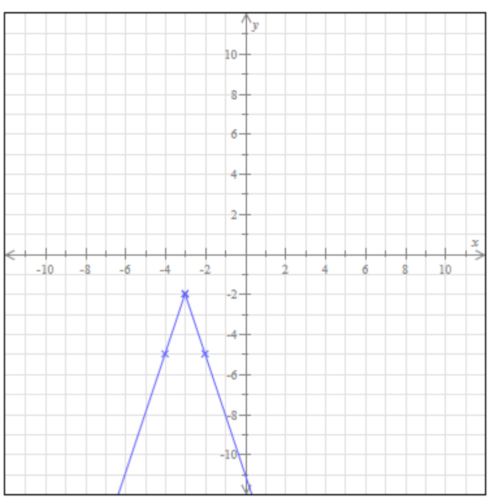
- ○ September to October
- ○ October to November
- ○ November to December
- ○ December to January
- ○ January to February

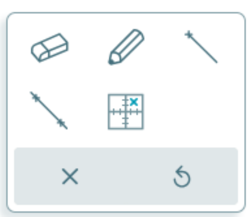


Graphing an absolute value equation in the plane: Advanced

Graph the equation.

$$y = -3\left|x+3\right| - 2$$



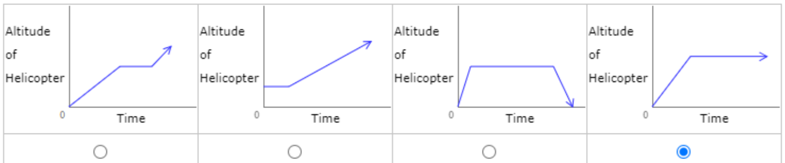


Help the learner to remember the concept of absolute value and invite him to make the table of values before graphing.

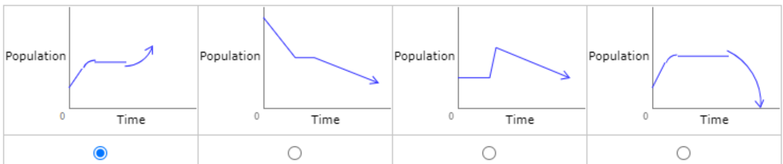
Choosing a graph to fit a narrative : Advanced

For each scenario below, choose the graph that gives the best representation.

(a) A police helicopter takes off straight up from the ground. It quickly ascends to several hundred meters above the ground. Then it flies straight ahead at a constant altitude for a few minutes. Without changing altitude, it turns and flies toward headquarters.



(b) The population of a large city grows very quickly for several years. Then the growth begins to slow, and the population remains constant for several years. After this, the population grows again, but slower than before.

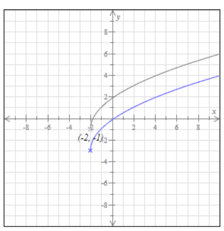


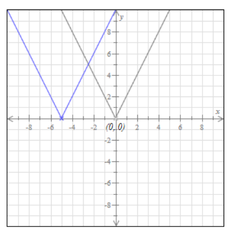
The learner must interpret the sentences to identify the behavior of the variables, point out the importance of locating each variable in the plane.

Translating the graph of a function: One step

Translate each graph as specified below.

(a) The graph of $y=f(x)$ is shown. Translate it to get the graph of $y=f(x)-2$.
(b) The graph of $y=g(x)$ is shown. Translate it to get the graph of $y=g(x+5)$.

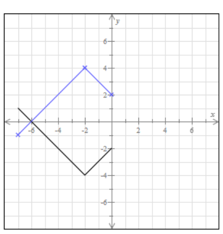




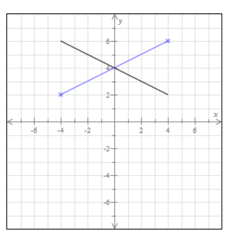
Emphasize with the learner that he must identify the form of the equation to know if it is a vertical translation or a horizontal translation.

Transforming the graph of a function by reflecting over an axis

(a) The graph of $y=f(x)$ is shown. Draw the graph of $y=-f(x)$.



(b) The graph of $y=g(x)$ is shown. Draw the graph of $y=g(-x)$.



In this exercise the learner must work with the coordinates of the ends of the graphs to achieve their reflection.

Notes for the teacher corresponding to the explanation of Topic 3

In this topic you will review with the learner the composition of functions, it is recommended that you start explaining with simple examples pointing out the importance of the order of the composition and demonstrating with practical exercises how this influences the result.

You can explain by diagramming the obtaining of the domain and range of a composite function.

Take up the concept of injective functions to specify that only this type of functions has an inverse function.

Establish the procedure for calculating the inverse of a function as a series of steps that the learner must follow, make sure that the learner understands the role of the dependent and independent variables at each point.

Develop an exercise where you show that when one function is inverse of another, the evaluations of the composition of the two functions in both directions have the same result *x*.

As for the domain and range of an inverse function, it is important that you start by demonstrating with the help of a relation of ordered pairs, how the domain of the function is the range of its inverse, just as the range of the function is equal to the domain of its inverse.

Notes for the teacher corresponding to Exercise 3

Before the explanation it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Introduction to the composition of two functions

The functions u and w are defined as follows.

$$u(x) = -4x - 3$$
$$w(x) = 2x - 2$$

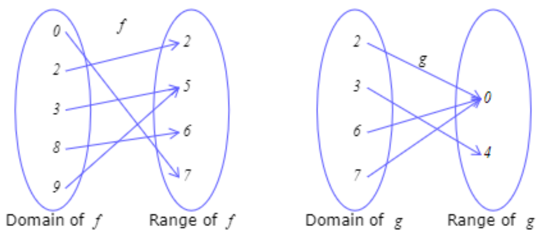
Find the value of $w(u(4))$.

$w(u(4)) = -40$

Point out that the order of the composition is decisive for the result.

Composition of two functions: Domain and range

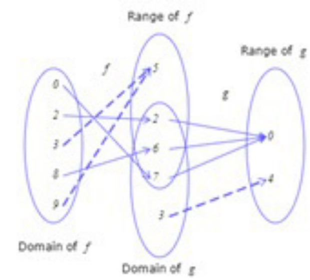
Two functions f and g are defined in the figure below.



Find the domain and range of the composition $g \circ f$. Write your answers in set notation.

Domain of $g \circ f$: $\{0, 2, 8\}$
Range of $g \circ f$: $\{0\}$

It is recommended that you use a graphical model to explain to the learner how to obtain the domain and range of the composite function:



Determining whether two functions are inverses of each other

For each pair of functions f and g below, find $f(g(x))$ and $g(f(x))$.
Then, determine whether f and g are inverses of each other.

Simplify your answers as much as possible.
(Assume that your expressions are defined for all x in the domain of the composition.
You do *not* have to indicate the domain.)

(a)	(b)
$f(x) = \frac{1}{6x}, \; x \neq 0$	$f(x) = \frac{1}{4x}, \; x \neq 0$
$g(x) = -6x$	$g(x) = \frac{1}{4x}, \; x \neq 0$
$f(g(x)) = -\frac{1}{36x}$	$f(g(x)) = x$
$g(f(x)) = -\frac{1}{x}$	$g(f(x)) = x$
○ f and g are inverses of each other	● f and g are inverses of each other
● f and g are *not* inverses of each other	○ f and g are *not* inverses of each other

In this exercise the learner must evaluate the compositions of the functions in both directions to check if they are inverse to each other.

Inverse functions: Problem type 1

The one-to-one functions g and h are defined as follows.

$$g(x) = -4x + 3$$

$$h = \{(-2, \; 2), \; (1, \; 6), \; (2, \; -5), \; (6, \; 9)\}$$

Find the following.

$g^{-1}(x) = \square$

$(g^{-1} \circ g)(3) = \square$

$h^{-1}(2) = \square$

In the second part of this exercise, demonstrate the procedure to obtain the inverse of the function and then evaluate the composition of both functions; point out to the learner that it is not necessary to do the whole process to obtain the result since the composition of a function with its inverse always results in an output value equal to the input value.

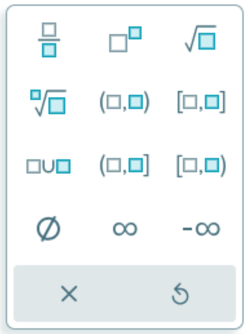
Inverse functions: Quadratic, square root

Consider the function $f(x) = \sqrt{x-2} + 10$ for the domain $[2, \infty)$.

Find $f^{-1}(x)$, where f^{-1} is the inverse of f.

Also state the domain of f^{-1} in interval notation.

$f^{-1}(x) = (x - 10)^2 + 2$ for the domain $[10, \infty)$



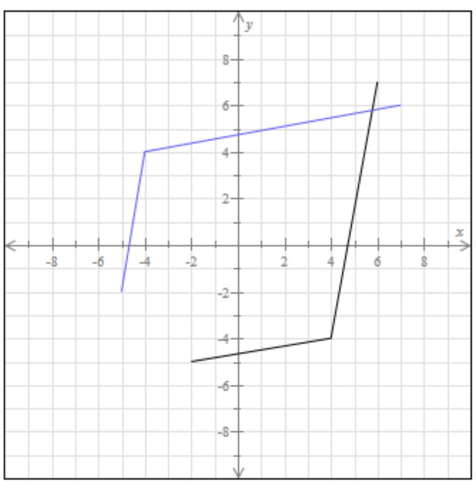
Detail and emphasize each of the steps to follow on the procedure to obtain the inverse of a function.

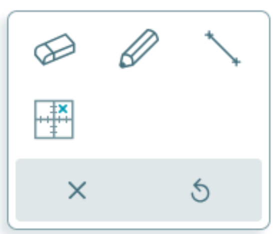
Guide the learner to obtain the range of the function based on the domain provided by the exercise and then point out that the range of the function corresponds to the domain of its inverse.

Graphing the inverse of a function given its graph

Below is the entire graph of function f.

Graph f^{-1}, the inverse of f.





Indicate to the learner that it is possible to plot the graph of the inverse of the shown function, working only with the coordinates of its ends and vertices.

Inverse functions: Problem type 2

The one-to-one function g is defined by

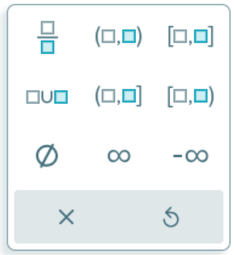
$$g(x) = \frac{2x}{5x-6}.$$

Find g^{-1}, the inverse of g. Then, give the domain and range of g^{-1} using interval notation.

$$g^{-1}(x) = \frac{-6x}{2-5x}$$

$$\text{Domain}\left(g^{-1}\right) = \left(-\infty, \frac{2}{5}\right) \cup \left(\frac{2}{5}, \infty\right)$$

$$\text{Range}\left(g^{-1}\right) = \left(-\infty, \frac{6}{5}\right) \cup \left(\frac{6}{5}, \infty\right)$$



Resume the steps in the procedure to define the inverse function.

The learner must find the domain of the function to define the range of its inverse and based on the range, calculate the domain of the inverse.

Notes for the teacher corresponding to the explanation of Topic 4

Begin by taking up the elements of the linear equation and helps the learner to remember how a straight line is drawn by its equation, as well as the behavior of this according to the value of its slope.

Emphasize the different forms of the equation of the line and how the learner will use each one depending on the available elements.

It is recommended that you demonstrate algebraically how (regardless of the form of the equation of the line) it is possible to transform it to the form $y = mx + b$, and once he gets this equation, the learner can identify the dependent and independent variables. In the same way, algebraically exemplify the procedure to get from any of the forms of the equation of the line to the form $Ax + By = C$ or $Ax + By + C = 0$, which corresponds to a polynomial of the first degree.

Explain the concept of the root of a function and point out how to obtain it for the linear function, as well as how the line can be drawn by knowing its root and the intersection on the *y-axis*.

Analyze with the student the symmetrical shape of the equation of the line and identify each of its elements.

Notes for the teacher corresponding to Exercise 4

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Identifying linear functions given ordered pairs

For each function, state whether it is linear.

Function 1	Function 2
$\{(0,-5),(4,-2),(8,1),(12,-4)\}$	$\{(-3,0),(-2,-2),(-1,-4),(0,-6)\}$
○ Linear ◉ Not linear	◉ Linear ○ Not linear

Function 3

x	y
5	2
7	6
9	10
11	14

◉ Linear
○ Not linear

Function 4

x	y
−4	−4
−3	−6
−2	−9
−1	−10

○ Linear
◉ Not linear

Explain to the learner that a function is linear if and only if a constant change in *x* produces a constant change in *y*.

Finding the slope and y-intercept of a line given its equation in the form Ax + By = C

Find the y-intercept and the slope of the line.

$$-5x+4y=-3$$

Write your answers in simplest form.

y-intercept: $-\frac{3}{4}$

slope: $\frac{5}{4}$

Undefined

Writing an equation in slope-intercept form given the slope and a point

A line passes through the point $(1,\,-1)$ and has a slope of -6.

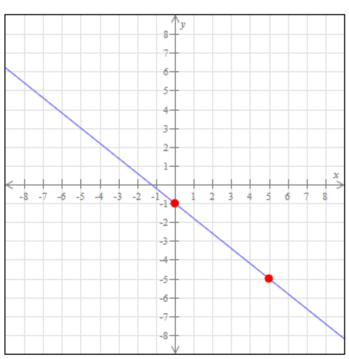
Write an equation in slope-intercept form for this line.

$y = -6x + 5$

In this type of exercises, the learner will practice the different forms of the equation of the line depending on the provided information.

Writing an equation of a line given the y-intercept and another point

Write an equation of the line below.



$y = -\frac{4}{5}x - 1$

Writing and evaluating a function that models a real-world situation: Advanced

The Sugar Sweet Company is going to transport its sugar to market. It will cost \$55 000 to rent trucks, and it will cost an additional \$2250 for each ton of sugar transported.

Let C represent the total cost (in pesos), and let S represent the amount of sugar (in tons) transported. Write an equation relating C to S. Then use this equation to find the total cost to transport 11 tons of sugar.

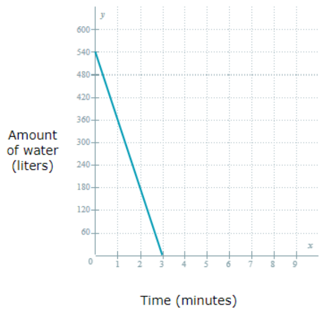
Equation: $C = 55000 + 2250S$

Total cost to transport 11 tons of sugar: \$79750

Highlight the importance of reading comprehension so that the learner can develop the equation requested in this exercise.

Finding the intercepts and rate of change given a graph of a linear function

Valentina is draining an aquarium. The graph shows the amount of water (in liters) in the aquarium versus time (in minutes).



(a) At what time does the amount of water in the aquarium reach 0 liters?

3 minutes

(b) Choose the statement that best describes how the time and amount of water are related. Then fill in the blank.

- (●) As time increases, the amount of water in the aquarium decreases.

  At what rate is the amount of water decreasing?

  180 liters per minute

- ( ) As time increases, the amount of water in the aquarium increases.

  At what rate is the amount of water increasing?

  ☐ liters per minute

Invite the learner to identify the dependent variable and the independent variable and to detect on which axis in the plane each one of them is located.

Interpreting the parameters of a linear function that models a real-world situation

A construction crew is lengthening a road. Let y represent the total length of the road (in kilometers). Let x represent the number of days the crew has worked. Suppose that x and y are related by the equation $y = 56 + 4x$.

Answer the questions below.

Note that a change can be an increase or a decrease.
For an increase, use a positive number. For a decrease, use a negative number.

What was the road's length when the crew started working?

56 kilometers

What is the change in the road's length per day?

4 kilometers

The learner will be able to identify the parameters of the equations that are presented (slope and y-intercept), to solve the exercise.

Identifying independent and dependent variables from equations or real-world situations

Answer the questions below.

(a) A function relates the input k to the output $n = 3k - 3$. Which is the independent variable for this function?	(●) ***k*** ( ) ***n***
(b) The cost of a want-ad in a newspaper increases with the length of the ad. For example, in one newspaper, a want-ad costs \$600 plus \$10 per word. Which is the independent variable in this relationship?	(●) length of want-ad ( ) cost of want-ad
(c) A company that makes plastic utensils is looking at how the price it charges for a knife affects the number of knives it sells per week. Which is the dependent variable?	( ) price of a knife (●) number of knives sold per week

Through reading comprehension, the learner must identify the dependent and independent variables.

Notes for the teacher corresponding to the explanation of Topic 5

Indicate the elements of a second-degree polynomial that represents a quadratic function: main term, main coefficient, and independent term.

Detail the elements of the graph that represent a quadratic function and explain the reason why a horizontal parabola cannot be considered as a function, for this you must show that, in the case of this type of equations, for each *x*, there are two *y* coordinates, which disqualifies it as a function.

Explain the characteristics of the graph of the quadratic function according to the value acquired by the main coefficient a, emphasize that the larger the value of a, the narrower the opening of the parabola and vice versa.

The learner must know that, to find the intersections on the *x-axis*, the equation must be solved in its general form by setting it to zero and solving for *x*.

It is recommended that you emphasize that, knowing the zeros of the function, it can be represented as the product of two factors and in the same way, when developing the multiplication of these two factors, a second-degree polynomial is obtained.

It is important that you develop an example that involves complex zeros and that you explain that when these types of values are presented for the roots of the function, it is indicative that the graph does not cross the *x-axis*.

Teach the learner the use of the discriminant to know if the graph of the function has intersections on the *x-axis* and how many.

Resume the procedure of completing the trinomial to obtain the standard form of the quadratic function.

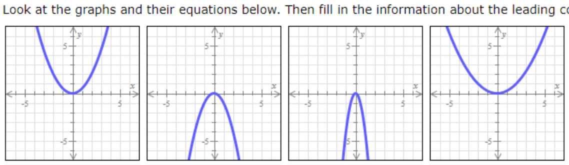
Notes for the teacher corresponding to Exercise 5

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

How the leading coefficient affects the shape of a parabola

Look at the graphs and their equations below. Then fill in the information about the leading coefficients A, B, C, and D.



$y=Ax^2$ $y=Bx^2$ $y=Cx^2$ $y=Dx^2$

	A	B	C	D
(a) For each coefficient, choose whether it is positive or negative	Positive	Negative	Negative	Positive
(b) Choose the coefficient closest to 0	○	○	○	◉
(c) Choose the coefficient with the least value	○	○	◉	○

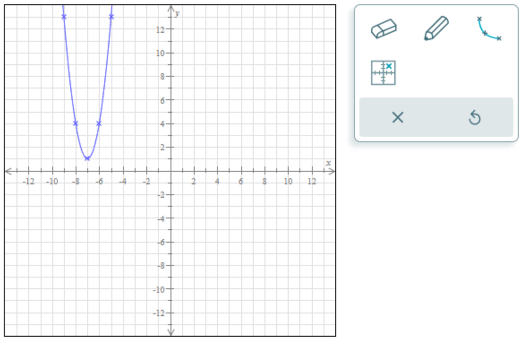
The learner must work with the characteristics of the graph of a quadratic function determined by the value of the main coefficient.

Graphing a parabola of the form $y = a(x-h)^2 + k$

Graph the parabola.

$$y = 3(x+7)^2 + 1$$

To graph the parabola, plot the vertex and four additional points, two on each side of the vertex. Then click on the graph icon.



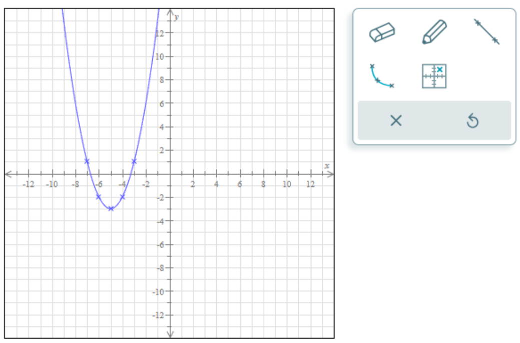
Explain that after locating the vertex coordinate, the learner must use at least one value to the left and one value to the right of the vertex to obtain two other coordinates and be able to plot the graph.

Graphing a parabola of the form $y = x^2 + bx + c$

Graph the parabola.

$$y = x^2 + 10x + 22$$

To graph the parabola, plot the vertex and four additional points, two on each side of the vertex. Then click on the graph icon.



The learner can use the formula to calculate the axis of symmetry in order to get the *x-coordinate* of the vertex and then obtain the *y-coordinate.*

Solving a quadratic equation by graphing

Solve the equation by graphing.

$$-x^2 + 8x - 16 = 0$$

First, graph the associated parabola by plotting the vertex and four additional points, two on each side of the vertex.

Then, use the graph to give the solution(s) to the equation.
If there is more than one solution, separate them with commas.

In this exercise, after graphing the function, the learner must locate the zeros or roots that are the solutions of the function.

Finding the zeros of a quadratic function given its equation

Find all the zeros of the quadratic function.

$$y = x^2 + 12x + 35$$

If there is more than one zero, separate them with commas.
If there are no zeros, click on "None".

zero(s): $-5, -7$

None

The learner must factor to calculate the zeros or roots of the function. Indicate that the quadratic formula may be used instead of factoring.

Writing a quadratic function given its zeros

Write a quadratic function h whose zeros are 5 and -6.

$h(x) = (x-5)(x+6)$

The learner must convert the zeros into factors and perform multiplication to obtain the second-degree equation representing the function.

Word problem involving the maximum or minimum of a quadratic function

The cost C (in pesos) of manufacturing x chairs at Joaquín's Furniture Factory is given by the function $C(x) = 0.6x^2 - 180x + 33\,449$. How many chairs must be made to minimize the cost?

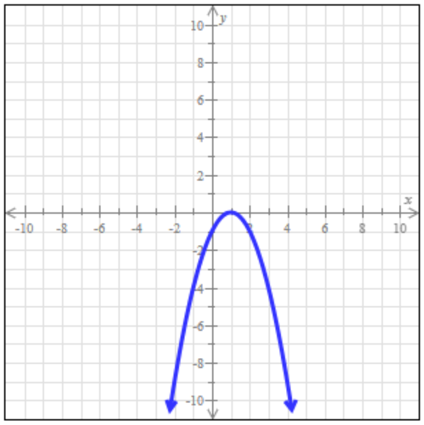
Do not round your answer.

Number of chairs: 150

Explain that once the vertex is found, the maximum or minimum of the function can be calculated and point out the importance of identifying whether to use the *x* or *y* coordinate of the vertex according to what is requested in the problem.

Domain and range from the graph of a quadratic function

The graph of a quadratic function with vertex $(1, 0)$ is shown in the figure below. Find the range and the domain.



Write the range and domain using interval notation.

(a) range:

(b) domain:

Identify in the graph the domain and range of the function.

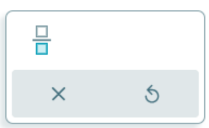
Discriminant of a quadratic equation

Compute the value of the discriminant and give the number of real solutions of the quadratic equation.

$$-2x^2+6x-1=0$$

Discriminant: 28

Number of real solutions: 2



The learner will use the discriminant formula.

Notes for the teacher corresponding to the explanation of Topic 6

It is important to make sure that the learner identifies a polynomial function from any other type of function.

Detail the characteristics of polynomial function graphs, guide the learner to visually identify from a graph, the degree of a function and, therefore, how many roots it has and of what type (real or complex).

Explain how the graphs of polynomial functions are influenced by the value of the main coefficient and exemplify graphically.

It is recommended that you do a review about factoring and how a polynomial can be represented as the multiplication of its factors.

Point out to the learner the difference between a factor and a root and how he can distinguish one from the other.

Address the synthetic division and emphasize that, if there is a missing term, it is necessary to respect its place in the dividend by using a zero; also point out that the obtained quotient is a degree less than the function before being divided and that the residue must be zero for the divisor to be a root of the function.

It is important that you present the quadratic formula again and explain that the values of *x* obtained by the formula are roots of the polynomial function.

Notes for the teacher corresponding to Exercise 6

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Identifying polynomial functions

For each function, determine whether it is a polynomial function.

Function	Is the function a polynomial?	
	Yes	No
(a) $f(x) = 7x^6$	◉	○
(b) $v(x) = -\frac{1}{x^2}$	○	○
(c) $u(x) = 3\sqrt{x} - 9x^4$	○	○
(d) $h(x) = x^8 - 7 + 3x^{-6}$	○	○

Finding a polynomial of a given degree with given zeros: Real zeros

Find a polynomial $f(x)$ of degree 5 that has the following zeros.

$3, -1, 4, -8, 6$

Leave your answer in factored form.

$f(x) = (x-3)(x+1)(x-4)(x+8)(x-6)$

Finding x- and y-intercepts given a polynomial function

Find all y-intercepts and x-intercepts of the graph of the function.

$$f(x) = x^3 - 3x^2 - x + 3$$

If there is more than one answer, separate them with commas.

Click on "None" if applicable.

y -intercept(s): 3

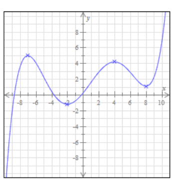
x -intercept(s): $3, 1, -1$

None

Explain that intersections with the *x-axis* are the roots of the function and that the graph of a function cannot have more than one intersection with the *y-axis* since each input *(x)* can have only one output *(y)* to be considered as a function.

Inferring properties of a polynomial function from its graph

Below is the graph of a polynomial function f with real coefficients. Use the graph to answer the following questions about f. All local extrema of f are shown in the graph.



(a) The function f is decreasing over which intervals? Choose all that apply.

- [ ] (-∞ , -7)
- [x] (-7, -2)
- [ ] (-2, 4)
- [ ] (-7, 4)
- [x] (4, 8)
- [ ] (8, ∞)

(b) The function f has local minima at which x-values? If there is more than one value, separate them with commas.

$-2, 8$

(c) What is the sign of the leading coefficient of f?

Positive

(d) Which of the following is a possibility for the degree of f? Choose all that apply.

- [ ] 4
- [x] 5
- [ ] 6
- [x] 7
- [ ] 8
- [x] 9

Synthetic division

Use synthetic division to find the quotient and remainder when $-2x^4 + 9x^3 + 5x^2 + 1$ is divided by $x-5$ by completing the parts below.

(a) Complete this synthetic division table.

5	−2	9	5	0	1
		−10	−5	0	0
	−2	−1	0	0	1

(b) Write your answer in the following form: Quotient $+ \frac{\text{Remainder}}{x-5}$.

$$\frac{-2x^4 + 9x^3 + 5x^2 + 1}{x-5} = -2x^3 - x^2 + \frac{1}{x-5}$$

Review with the learner how to identify each element of the synthetic division and how to organize them in their answers.

Using the remainder theorem to evaluate a polynomial

Use the remainder theorem to find $P(-3)$ for $P(x) = -2x^4 - 5x^3 + 5x^2 - 3$.

Specifically, give the quotient and the remainder for the associated division and the value of $P(-3)$.

Quotient = $-2x^3 + x^2 + 2x - 6$

Remainder = 15

$P(-3)$ = 15

Point out to the learner that, when using synthetic division, the reminder is the same as the result of evaluating the function with the value of the divisor.

The Factor Theorem

Use the Factor Theorem to determine whether $x-1$ is a factor of $P(x) = -x^4 + x^3 + 5x^2 - 7$.

Specifically, evaluate P at the proper value, and then determine whether $x-1$ is a factor.

$P(1)$ = -2

○ $x - 1$ is a factor of $P(x)$

◉ $x - 1$ is *not* a factor of $P(x)$

Using a given zero to write a polynomial as a product of linear factors: Real zeros

For the polynomial below, 2 is a zero.

$$g(x) = x^3 - 8x^2 + 19x - 14$$

Express $g(x)$ as a product of linear factors.

$g(x) = (x-2)(x-(3+\sqrt{2}))(x-(3-\sqrt{2}))$

Point out the difference between factor and zero (root).

Finding all possible rational zeros using the rational zeros theorem: Problem type 1

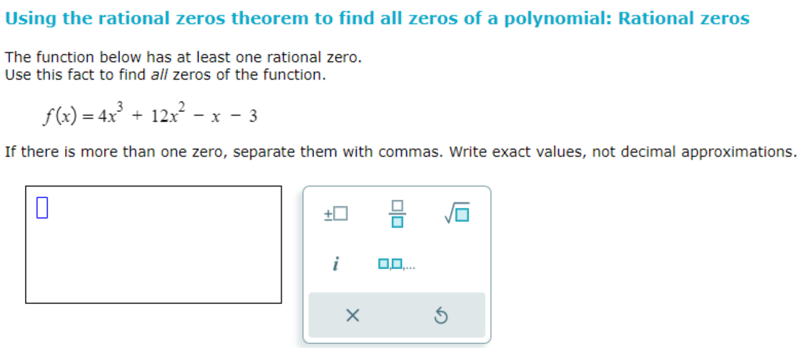
Use the rational zeros theorem to list all possible rational zeros of the following.

$$f(x) = -3x^4 + 7x^3 + 6x^2 + 9x + 5$$

Be sure that no value in your list appears more than once.

$1, -1, 5, -5, \frac{1}{3}, -\frac{1}{3}, \frac{5}{3}, -\frac{5}{3}$

The learner should only list the possible rational zeros.



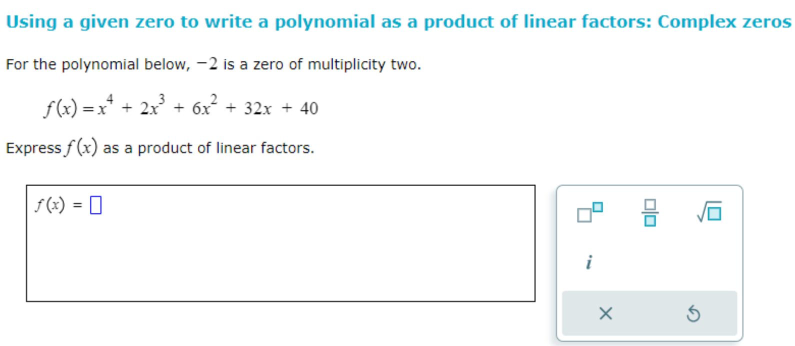
Using the rational zeros theorem to find all zeros of a polynomial: Rational zeros

The function below has at least one rational zero.
Use this fact to find *all* zeros of the function.

$$f(x) = 4x^3 + 12x^2 - x - 3$$

If there is more than one zero, separate them with commas. Write exact values, not decimal approximations.

Indicate that after having obtained the list of possible zeros or roots, the learner must use the synthetic division to review each possible root until he finds one that has a zero as residue and, therefore, it is verified that it is a root.



Using a given zero to write a polynomial as a product of linear factors: Complex zeros

For the polynomial below, -2 is a zero of multiplicity two.

$$f(x) = x^4 + 2x^3 + 6x^2 + 32x + 40$$

Express $f(x)$ as a product of linear factors.

Take up the concept of complex numbers and the powers of *i*.
During the explanation use the quadratic formula on the quotient to obtain the rest of the zeros.

Notes for the teacher corresponding to the explanation of Topic 7

Explain by example how to differentiate a rational function and point out that the values of the function do not allow a zero in the denominator since it would cause an indetermination. Work with the concepts of domain, range, and interval notation.

Point out the different types of asymptotes and how to calculate them, emphasize that an asymptote should not always be present and how to determine this situation. Work with the learner on an example of an oblique asymptote and help him identify its equation as a line equation of the form *y = mx + b.*

Resume the process of long division, indicate to the learner each of its elements and the procedure to calculate it.

Notes for the teacher corresponding to Exercise 7

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Domain of a rational function

The function h is defined below.

$$h(x) = \frac{x-4}{x^2+6x+9}$$

Find all values of x that are NOT in the domain of h.
If there is more than one value, separate them with commas.

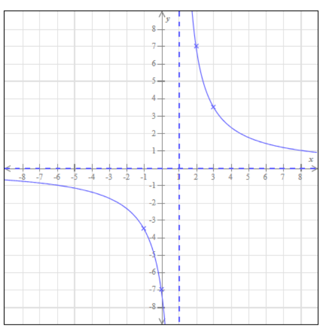
$x = -3$

You can give the option to use factoring or the quadratic formula to obtain the values that should be omitted in the domain and avoid a division by zero.

Graphing a rational function: Constant over linear

Graph the rational function $f(x) = \frac{7}{x-1}$.

To graph the function, draw the horizontal and vertical asymptotes (if any) and plot at least two points on each piece of the graph. Then click on the graph icon.



The learner must determine the asymptotes (if they exist), and then use values of x to the left and right of the vertical asymptote to tabulate and obtain at least two points on each side of the asymptote in order to graph the function.

Finding x- and y-intercepts of the graph of a nonlinear equation

Find the y-intercept(s) and x-intercept(s) of the graph of the following.

$$x^2 + y = 25$$

If there is more than one answer, separate them with commas.
Click on "None" if applicable.

y-intercept(s): 25

x-intercept(s): 5, −5

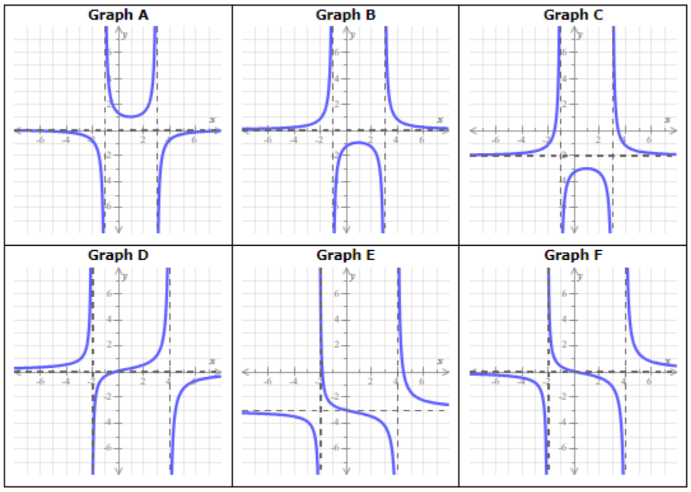
Matching graphs with rational functions: Two vertical asymptotes

Consider the following rational functions.

$$f(x) = \frac{4}{x^2 - 2x - 3}$$

$$g(x) = \frac{2x}{x^2 - 2x - 8}$$

Choose the graph of each function from the choices below.



Which is the graph of $f(x) = \frac{4}{x^2 - 2x - 3}$?	Graph B
Which is the graph of $g(x) = \frac{2x}{x^2 - 2x - 8}$?	Graph F

Explain to the learner how to make a table of signs for each interval of the function and based on these signs, the learner will be able to determine the direction of each stroke as it approaches the asymptotes.

Notes for the teacher corresponding to the explanation of Topic 8

Take step functions and present them as an example of a piecewise function. It is recommended to use a real-life example that is familiar to the learner to exemplify the piecewise functions.

Point out that a function defined by parts is a set of rules that correspond only to a certain interval of the domain of the function.

In the graph of the piecewise function, you must explain the meaning of the empty points and the full points and how they relate to the interval notation.

Emphasize that each range of values in the domain has its own graphical representation, which depends on the rule that corresponds to it.

Try to use a graph to point out to the learner the domain and range of a piecewise function.

Notes for the teacher corresponding to Exercise 8

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Evaluating a piecewise-defined function

Suppose that the function h is defined, for all real numbers, as follows.

$$h(x) = \begin{cases} \frac{3}{4}x+1 & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$$

Find $h(-5)$, $h(-2)$, and $h(3)$.

$h(-5) = \square$

$h(-2) = \square$

$h(3) = \square$

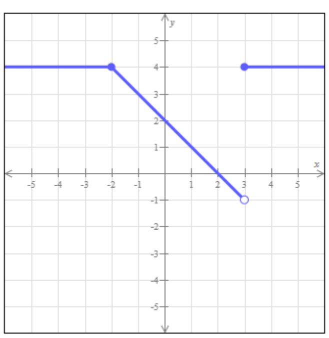
Explain that the rule to use, in order to calculate the output value, depends on the input value.

Introduction to graphing a piecewise-defined function involving lines with non-zero slope

Suppose that the function g is defined, for all real numbers, as follows.

$$g(x) = \begin{cases} 4 & \text{if } x < -2 \\ -x+2 & \text{if } -2 \leq x < 3 \\ 4 & \text{if } x \geq 3 \end{cases}$$

Graph the function g.



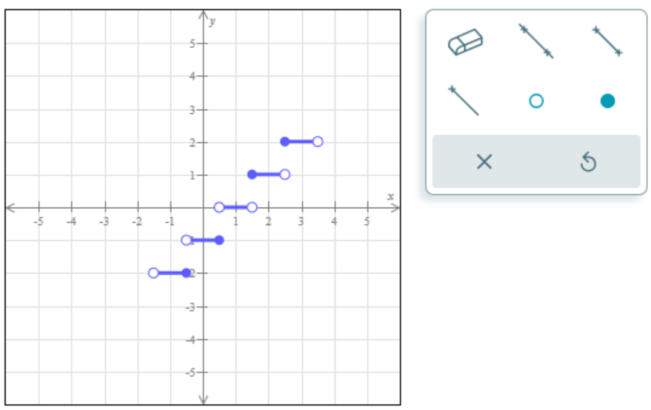
Make sure the learner identifies the values that correspond to each interval.

Graphing a piecewise-defined function

Suppose that the function g is defined on the interval $(-1.5, -2.5)$ as follows.

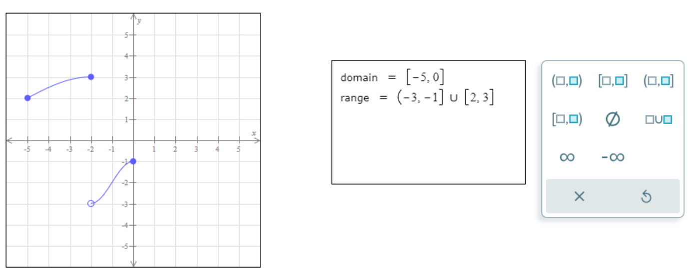
$$g(x)=\begin{cases}-2 & \text{if } -1.5<x\le -0.5\\ -1 & \text{if } -0.5<x\le 0.5\\ 0 & \text{if } 0.5<x<1.5\\ 1 & \text{if } 1.5\le x<2.5\\ 2 & \text{if } 2.5\le x<3.5\end{cases}$$

Graph the function g.



Domain and range from the graph of a piecewise function

The entire graph of the function f is shown in the figure below.
Write the domain and range of f as intervals or unions of intervals.



The learner must identify the intervals defined in each of the strokes of the graph and use the interval notation to answer.

Notes for the teacher corresponding to the explanation of Topic 9

Present the form of the exponential function and each of the elements that form it, guide the learner to identify an exponential function.

It is important for the learner to review the rules of exponents to facilitate the evaluation of exponential functions.

Explain to the learner the equivalence of the Euler number, how it gives rise to the natural exponential function and its role in the formula of population growth.

Make sure the learner identifies the graph of an exponential function and knows how the value of the base influences the behavior of the graph.

Analyze the characteristics of the graphs of exponential functions, including their range and domain and contrast two examples that allow the learner to identify their differences.

Guide the learner to recognize the algebraic form of an exponential function that moves vertically or horizontally and show the translation graphically.

Review the properties of exponential functions, including the relation among successive values of *y*.

Notes for the teacher corresponding to Exercise 9

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Solving an exponential equation by finding common bases: Linear and quadratic exponents

Solve for x.

$$9^{x^2-9x-14} = 81^{8-4x}$$

If there is more than one solution, separate them with commas.

$x = 6, -5$

No solution

The learner will seek to equalize the bases on both sides of the equation in order to match the resulting exponents and determine the value of the variable.

Table for an exponential function

The function f is defined by the following rule.

$$f(x) = 10^x$$

Find $f(x)$ for each x-value in the table.

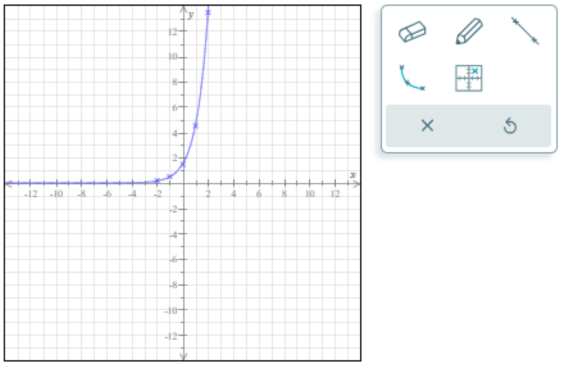
x	$f(x)$
-1	$\frac{1}{10}$
0	1
1	10
2	100
3	1000

The learner must apply the rules of the exponents to evaluate the function.

Graphing an exponential function: f(x) = a(b)^x

Graph the exponential function $g(x) = \frac{3}{2}(3)^x$.

To graph the function, plot the points on the graph with x-values -2, -1, 0, 1, and 2, and then click on the graph icon.



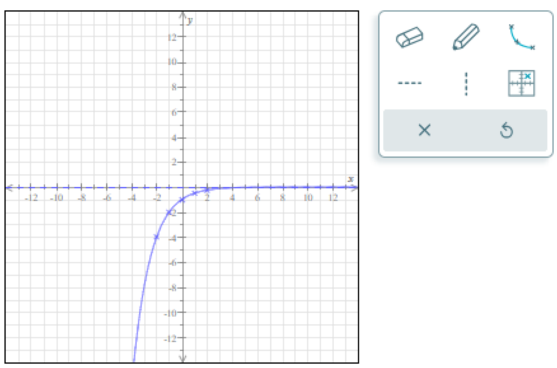
This exercise provides the input values for calculating the coordinates that will serve as a guide to plot the function.

Graphing an exponential function and its asymptote: f(x)=b^-x or f(x)=-b^ax

Graph the exponential function $f(x) = -\left(\frac{1}{2}\right)^x$.

To graph the function, plot the points on the graph with x-values -2, -1, 0, 1, and 2.

Next draw the asymptote(s), and then click on the graph button.

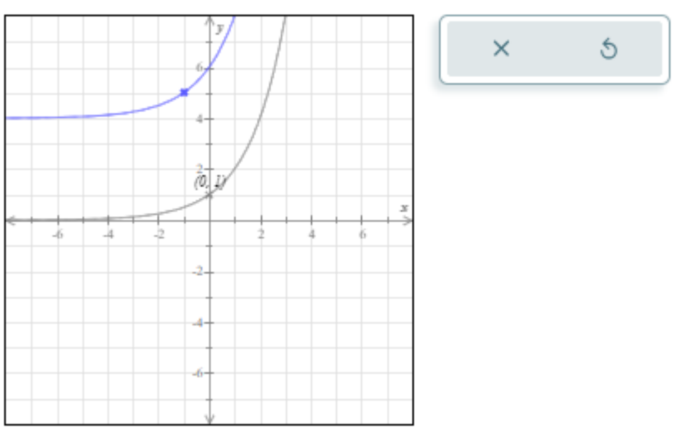


The learner must identify the type of function to know the shape of its graph and determine the asymptote after evaluating the function with the input values given in the exercise.

Translating the graph of an exponential function

Below is the graph of $y = 2^x$.

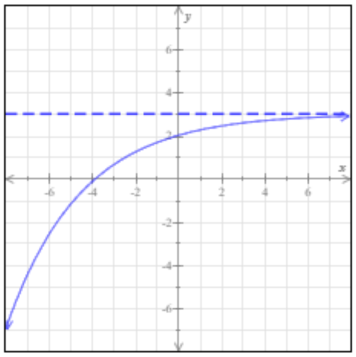
Translate it to become the graph of $y = 2^{x+1} + 4$.



Knowing the initial function and given the shape of the function after it has been translated, the learner can identify the type of translation and locate two points on the graph to plot it.

Finding domain and range from the graph of an exponential function

The graph of an exponential function is shown in the figure below.
The horizontal asymptote is shown as a dashed line.
Find the range and the domain.



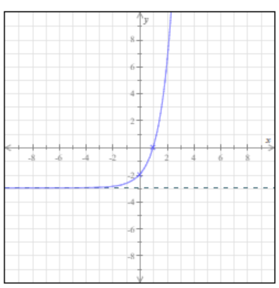
Write your answers as inequalities, using x or y as appropriate.
Or, you may instead click on "Empty set" or "All reals" as the answer.

range: $y < 3$

domain: All reals

The graph, domain, and range of an exponential function

Graph the function $g(x) = 3^x - 3$ and give its domain and range using interval notation.



Domain: $(-\infty, \infty)$

Range: $(-3, \infty)$

The learner must plot the function to determine its domain and range.

Finding the initial amount in a word problem on continuous compound interest

Mr. and Mrs. Pino hope to send their son to college in fourteen years. How much money should they invest now at an interest rate of 8.5% per year, compounded *continuously*, in order to be able to contribute $\$8500$ to his education?

Do not round any intermediate computations, and round your answer to the nearest cent.

$\$2585.88$

Finding the final amount in a word problem on continuous exponential growth or decay

The mass of a radioactive substance follows a *continuous exponential decay model*. A sample of this radioactive substance has an initial mass of 9749 kg and decreases continuously at a relative rate of 4% per day. Find the mass of the sample after five days.

Do not round any intermediate computations, and round your answer to the nearest tenth.

7981.8 kg

Applications of exponential functions in real-life situations.

Finding the final amount in a word problem on continuous exponential growth or decay

The number of bacteria in a certain population is predicted to increase according to a *continuous exponential growth model*, at a relative rate of 3% per hour. Suppose that a sample culture has an initial population of 934 bacteria. Find the predicted population after four hours.

Do not round any intermediate computations, and round your answer to the nearest tenth.

1053.1 bacteria

Identifying linear, quadratic, and exponential functions given ordered pairs

For each function, state whether it is linear, quadratic, or exponential.

Function 1

x	y
3	2
4	8
5	32
6	128
7	512

○ Linear
○ Quadratic
◉ Exponential
○ None of the above

Function 2

x	y
5	-51
6	-36
7	-25
8	-18
9	-15

○ Linear
◉ Quadratic
○ Exponential
○ None of the above

Function 3

x	y
2	6
3	17
4	28
5	39
6	50

◉ Linear
○ Quadratic
○ Exponential
○ None of the above

According to the relation between successive output values, the learner must identify what type of function it is.

Notes for the teacher corresponding to the explanation of Topic 10

Emphasize the relation between logarithmic equations and exponential equations, and present to the learner the classification of logarithms according to the base each one of them has, guiding them to differentiate their nomenclature.

It is recommended to work conversion exercises between logarithmic expressions and exponential expressions and vice versa, using different bases for logarithms.

Explain the graphs of logarithmic functions by means of an example that demonstrates how these graphs reflect the graphs of their corresponding exponential functions. Particularly point out the difference between the asymptotes of each type of function.

Point out to the learner that he can predict the shape and direction of the graph of a logarithmic function depending on the value of the variable b in the function and that there are three key points that can help him trace the graph.

Ask the learner to check on his calculator if he has the option to calculate a logarithm of any base, explain how to use the base change formula.

Notes for the teacher corresponding to Exercise 10

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Converting between logarithmic and exponential equations

Rewrite each equation as requested.

(a) Rewrite as an exponential equation.

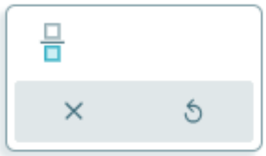
$$\log_8 1 = 0$$

(b) Rewrite as a logarithmic equation.

$$3^4 = 81$$

(a) $8^0 = 1$

(b) $\log_3 81 = 4$

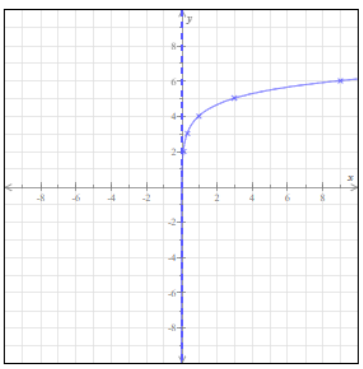


Suggest to the learner to first identify each of the elements of the expression that he will transform.

Graphing a logarithmic function: Basic

Graph $g(x) = 4 + \log_3 x$.

To graph the function, plot at least two points on the graph, draw all asymptotes, and then click on the graph icon.



In this exercise, when tabulating, it is recommended to use values of x that are powers of the base of the logarithm, which would facilitate the calculation of y or *f(x).*

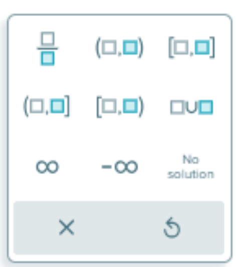
Domain of a logarithmic function: Advanced

Find the domain of the function.

$$f(x) = \log\left(x^2 - 4\right)$$

Write your answer as an interval or union of intervals.

Domain: $(-\infty, -2) \cup (2, \infty)$



Note that the variable *x* is located within a square root, therefore, that expression cannot be a value less than zero.

Basic properties of logarithms

Fill in the missing values to make the equations true.

$$\log_4 5 + \log_4 9 = \log_4 45$$

$$\log_3 2 - \log_3 7 = \log_3 \frac{2}{7}$$

$$\log_2 \frac{1}{9} = -2 \log_2 3$$

Expanding a logarithmic expression: Problem type 1

Use the properties of logarithms to expand $\log \frac{y^8}{z}$.

Each logarithm in your answer should involve only one variable.
Assume that all variables are positive.

$$\log \frac{y^8}{z} = 8 \log y - \log z$$

Expanding a logarithmic expression: Problem type 3

Use the properties of logarithms to expand the following expression.

$$\log\left(\frac{4x^5}{(6+x)^3}\right)$$

Your answer should not have radicals or exponents.

You may assume that all variables are positive.

$$\log\left(\frac{4x^5}{(6+x)^3}\right) = \log 4 + 5 \log x - 3 \log(6+x)$$

The learner must use the properties of logarithms to solve these exercises.

Using properties of logarithms to evaluate expressions

Use the properties of logarithms to evaluate each of the following expressions.

(a) $\ln e^5 - 2 \ln e^6 = -7$

(b) $\log_{14} 7 + \log_{14} 2 = 1$

Writing an expression as a single logarithm

Write the expression as a single logarithm.

$$4\log_8(4y+1)+\frac{1}{5}\log_8(y+4)$$

$\log_8\left((4y+1)^4(y+4)^{\frac{1}{5}}\right)$

Change of base for logarithms: Problem type 2

Use the base change formula.

Consider the equation

$$\log_5 14^{x+3}=20.$$

Find the value of x. Round your answer to 3 decimal places.

$x = 9.197$

Finding the rate or time in a word problem on exponential growth or decay

Suppose that the number of bacteria in a certain population increases according to a *continuous exponential growth model*. A sample of 3000 bacteria selected from this population reached the size of 3322 bacteria in four hours. Find the hourly growth rate parameter.

Note: This is a *continuous* exponential growth model.

Write your answer as a percentage. Do not round any intermediate computations, and round your percentage to the nearest hundredth.

2.55 %

The learner must use the formula of continuous exponential growth or decrease, converting the resulting expression to a natural logarithm in order to solve it.

Finding the time given an exponential function with base e that models a real-world situation

The number of milligrams $D(h)$ of a certain drug that is in a patient's bloodstream h hours after the drug is injected is given by the following function.

$$D(h)=45e^{-0.15h}$$

When the number of milligrams reaches 17, the drug is to be injected again. How much time is needed between injections?

Round your answer to the nearest tenth, and do not round any intermediate computations.

6.5 hours

The solution requires converting the exponential function to a logarithmic expression.

Notes for the teacher corresponding to the explanation of Topic 11

Begin by remembering the concepts of trigonometric ratios, then guide the learner to identify a trigonometric function.

Explain the particular characteristics of the sine and cosine functions, detail each of the elements of their algebraic expressions and what they represent within the sinusoidal graphs. It is recommended to graphically present each of the parameters both in a graph of the sine function and in a graph of the cosine function.

Point out how the sign of the coefficient a in the trigonometric function affects the shape that the graph acquires both in the sine and cosine functions and emphasize the importance for the learner to know what shape the graph will have before graphing, because this will facilitate its plotting.

Show the sine and cosine graphs without any displacement and allow the learner to locate the roots, the intersection on the *y-axis* and the minimum and maximum points, explain how these 5 points will be decisive for the drawing of the graph.

Then contrast with examples of sine and cosine graphs that present horizontal and vertical displacements, guide the learner to locate the roots, the intersection on the y-axis and the minimum and maximum points.

Explain how to obtain the coordinates of the roots and the minimum and maximum points of the graph using the parameters of the trigonometric function.

In the same way, describe, by means of an example, how to obtain the trigonometric function from its graph.

Notes for the teacher corresponding to Exercise 11

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

In the activities corresponding to the topic in Aleks you will find exercises such as:

Amplitude and period of sine and cosine functions

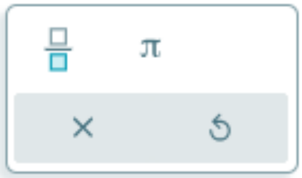
Find the period and amplitude of the function.

$$y = 2\,\text{sen}\,4x$$

Give the exact values, not decimal approximations.

Period: $\frac{\pi}{2}$

Amplitude: 2



The learner must identify the parameters of the trigonometric function in order to calculate the requested data.

Amplitude, period, and phase shift of sine and cosine functions

Find the amplitude, phase shift, and period of the function.

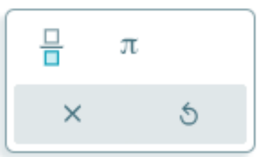
$$y = 3 + 4\cos(2\pi x - \pi)$$

Give the exact values, not decimal approximations.

Amplitude: 4

Phase shift: $\frac{1}{2}$

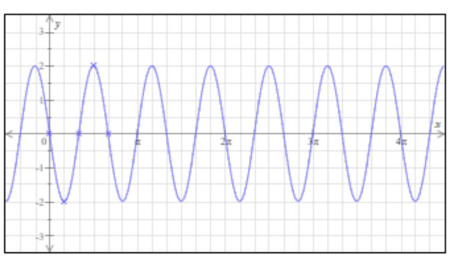
Period: 1



Inform the learner that phase shift refers to horizontal shift.

Sketching the graph of y= *a* sen(*bx*) or y= *a* cos(*bx*)

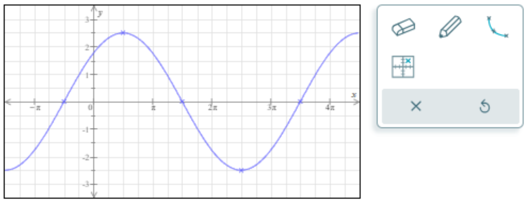
Graph the function $y = -2\,\text{sen}\,3x$.
To draw the graph, plot all x-intercepts, minima, and maxima within one period. Then click on the graph icon.



Sketching the graph of y= a sen(bx+c) or y= a cos(bx+c)

Graph the function $y = \frac{5}{2}\,\text{sen}\left(\frac{1}{2}x + \frac{\pi}{4}\right)$.

To draw the graph, plot all x-intercepts, minima, and maxima within one period. Then click on the graph icon.



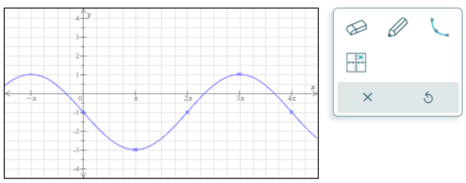
Guide the learner to identify the parameters of the function so that before tracing the graph he knows if the graph presents some type of displacement.

Sketching the graph of y= a sen(bx)+ d or y= a cos(bx)+ d

Graph the trigonometric function.

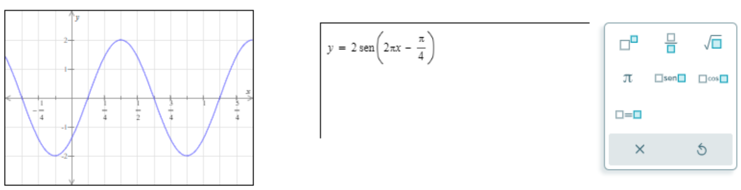
$$y = -2\,\text{sen}\left(\frac{1}{2}x\right) - 1$$

Plot all points corresponding to minima and maxima within one cycle. Within that cycle, also plot all points along the "midline" (points whose y-coordinates are midway between the function's minimum and maximum values). Then click on the graph-a-function button.



Writing the equation of a sine or cosine function given its graph: Problem type 2

Write the equation of a sine or cosine function to describe the graph.



Explain to the learner that depending on the cycle he selects on the graph; he could obtain several equations of the function described by the same graph.

Notes for the teacher corresponding to the explanation of Topic 12

Informally, explain that a continuous function is one that does not present any jump or interruption in its stroke.

Point out that graphs can be continuous in reference to only a certain interval.
Return to the concept of limits and explain each of the three conditions of continuous functions, it is recommended that you rely on graphs that exemplify each condition.

In the same way, algebraically prove the three conditions of continuity.

Work with examples of functions whose graphs are discontinuous and explain at what point and under what condition the discontinuity occurs.

Notes for the teacher corresponding to Exercise 12

Before the explanation, it is necessary for the learner to review the previous activity as a preparation of the topic.

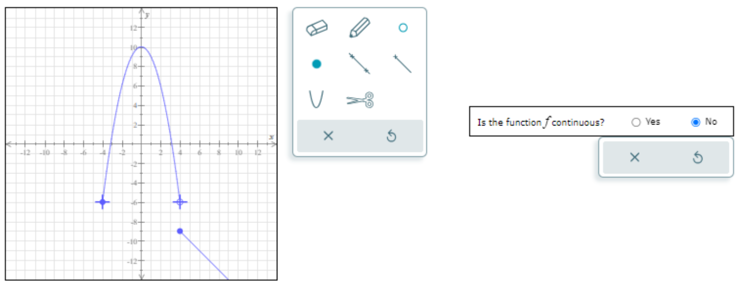
In the activities corresponding to the topic in Aleks you will find exercises such as:

Graphing a piecewise-defined function: Problem type 3

Suppose that the function f is defined for all real numbers as follows.

$$f(x)=\begin{cases} -x^2+10 & \text{if } -4\leq x<4 \\ -5-x & \text{if } x\geq 4 \end{cases}$$

Graph the function f. Then determine whether or not the function is continuous.



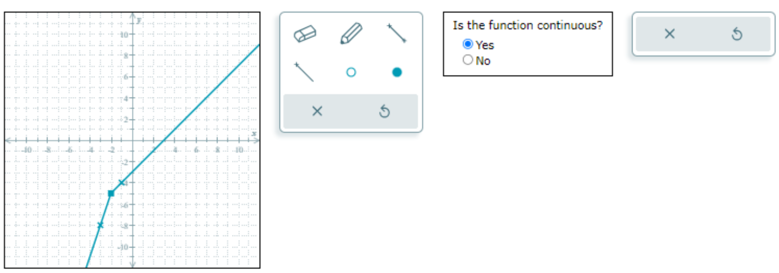
The learner must draw the graph and define by observation whether it is a continuous or discontinuous graph.

Graphing a piecewise-defined function: Problem type 2

Suppose that the function f is defined, for all real numbers, as follows.

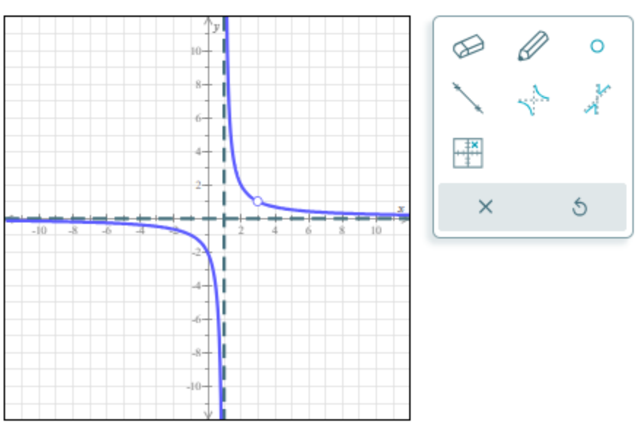
$$f(x)=\begin{cases} 3x+1 & \text{if } x<-2 \\ x-3 & \text{if } x\geq -2 \end{cases}$$

Graph the function f. Then determine whether or not the function is continuous.



Graphing rational functions with holes

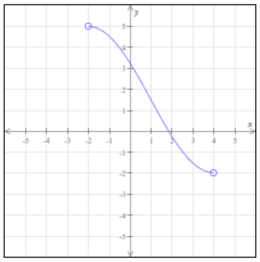
Graph the rational function $f(x)=\dfrac{2x-6}{x^2-4x+3}$.



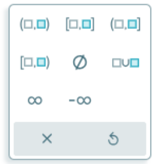
Tell the learner that the holes in the graph of the function are the points of discontinuity.

Domain and range from the graph of a continuous function

The entire graph of the function g is shown in the figure below.
Write the domain and range of g using interval notation.



domain= $(-2, 4)$
range= $(-2, 5)$



Restriction on a variable in a denominator: Quadratic

Find all excluded values for the expression.
That is, find all values of w for which the expression is undefined.

$$\frac{w-6}{w^2-36}$$

If there is more than one value, separate them with commas.

$w = 6, -6$

Point out to the learner that the excluded points in the expression are the points where the discontinuity occurs.